

3. Abstracts

On quandle homology groups of finite quandles

Yongju Bae

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Abstract: A quandle is a set Q together with a binary operation which satisfies three properties. Carter-Jelsovsky-Kamada-Saito defined the quandle homology group and the quandle cohomology group of a quandle Q and calculated the first quandle homology group. In this talk, we will calculate the second quandle homology group of finite quandles and try to get related results.

The Braid Index of Complicated DNA Polyhedral Links

Xiaosheng Cheng

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Abstract: The goal of this paper is to determine the braid index of two types of complicated DNA polyhedral links introduced by chemists and biologists in recent years. We shall study it in a more broad context and actually consider so-called Jaegers links (more general Traldis links) which contain, as special cases, both four types of simple polyhedral links whose braid indexes have been determined and the above two types of complicated DNA polyhedral links. Denote by $b(L)$ and $c(L)$ the braid index and crossing number of an oriented link L , respectively. Roughly speaking, in this paper, we prove that $b(L) = c(L)/2 + 1$ for any link L in a family including Jaegers links and contained in Traldis links, which is obtained by combining the MFW inequality and an Ohyamas result on upper bound of the braid index. Our result may be used to characterize and analyze the structure and complexity of DNA polyhedra and entanglement in biopolymers.

Chromatic Equivalence Classes of Graphs

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Abstract: Let G be a graph and let $P(G, t)$ denote its chromatic polynomial. Let $\mathcal{C}(G)$ denote the set of all graphs having the same chromatic polynomial with that of G . $\mathcal{C}(G)$ is known as the chromatic equivalence class of G . G is said to be chromatically unique if $\mathcal{C}(G) = \{G\}$. Chromatically unique graphs have been quite extensively studied (see the book “Dong, Koh and Teo, Chromatic Polynomials and Chromaticity of Graphs, World Scientific, 2005” for detailed account). In this lecture, we focus our attention on chromatic equivalence classes of graphs.

Ropelengths of Knots

Yuanan Diao

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Abstract: In this talk I will introduce the concept of the thickness of a knot and the ropelength of a knot. I will then give a brief review of the various results concerning the lower and upper bounds of the ropelengths in general and for some special families of knots.

On flow polynomials with real roots only

Fengming Dong

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Abstract: It is well known that for a chordal graph G , the chromatic polynomial $P(G, t)$ of G has integer roots only. But the converse statement does not hold. However, the converse statement is really true if G is planar. This result has been further extended to flow polynomials of an arbitrary graph by Kung and Royle: if the flow polynomial $F(G, t)$ of a graph G has integer roots only, then G is the dual of some plane and chordal graph. There exist non-planar graphs G such that all roots of $P(G, t)$ are real but some of them are not integers. However, so far no planar graphs with such property have been found. We guess that for a planar graph G , if all roots of $P(G, t)$ are real, then all roots of $P(G, t)$ are integers. More generally, we conjecture that for an arbitrary graph G , if all roots of $F(G, t)$ are real, then all roots of $F(G, t)$ are integers. This conjecture has been verified for some cases, one of which is that $F(G, t)$ has no real roots in the interval $(1, 2)$. In this talk, we will present some results on the study of this conjecture.

Edge spanning tree entropy of planar graphs, determinant density of links and the Kenyon's conjecture

Jun Ge

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Abstract: In this talk we focus on edge spanning tree entropy of planar graphs, determinant density of links and the Kenyon's conjecture. We point out that a Stoimenow's result can give the best known upper bound for edge spanning tree entropy of planar graphs and determinant density of links. We verify the Kenyon's conjecture for links with at most 47 crossings, and equivalently, for planar graphs with at most 47 edges. Special classes such as torus links, pretzel links, generalized theta graphs, outerplanar graphs, Archimedean lattices and Lave lattices, 3-regular graphs are also verified. We also study the forbidden structures in minimal counterexamples.

Some extreme coefficients of the Tutte polynomial of graphs

Helin Gong*

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Abstract: The Tutte polynomial is a well-studied invariant in combinatorics and also used to study knots in Knot theory and spin models in statistical physics. The Tutte polynomial of a graph G evaluates several well-known isomorphic invariants such as the chromatic polynomial and the flow polynomial of G . In the present paper, we shall give an expression for the flow polynomial of G according to a partial order relation defined on bridgeless spanning subgraphs of G , then by applying the convolution formula of the Tutte polynomial we derive several extreme coefficients of the Tutte polynomial.

Dominance graphs of Efron's nontransitive coins and semiacyclic tournaments

Gabor Hetyei

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Abstract: We provide necessary and sufficient conditions for a tournament to be the dominance graph of a set of unfair coins. We completely characterize the tournaments that are dominance graphs of sets of coins in which each coin displays its larger side with greater

probability. The class of these tournaments coincides with the class of tournaments whose vertices can be numbered in a way that makes them semiacyclic, as defined by Postnikov and Stanley. We provide an example of a tournament on nine vertices that can not be made semiacyclic, yet it may be represented as a dominance graph of coins, if we also allow coins that display their smaller side with greater probability. We conclude with an example of a tournament with 81 vertices that is not the dominance graph of any system of coins.

Elastic Network Modeling of Protein Cages

Guang Hu

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Abstract: With the rapid development of structural nanotechnology, natural proteins have become versatile building blocks for assembling well-ordered structures. Protein cages constitute the basis for molecular machines, and the understanding of their allosteric properties and functions is just at the beginning. Yet, computational methods to model their structures and dynamics remain to be established. In this work, group theory and Anisotropic Network Model are used to predict and analyze collective motions, and Markovian stochastic analysis coupled with Gaussian Network Model are used to study the signal-processing properties, of two artificial protein cages. First, our study indicates that collective motions of the protein tetrahedron are the combinations of face motions and of the protein cube are the combinations of vertex and face motion. The face bending and face breathing were suggested as two important motions, as they are likely to be involved in drug delivery. Second, key residues acting as sensors and effectors and communication pathways both existing in intra- and inter-monomers were identified. The analysis provides evidence for the critical roles of the α -linker and the residue pair Thr22 - Ile390 in establishing the communication between vertex and edge domains in the protein tetrahedron, as well as the importance of interface residues in mediating signals between two monomers in the protein cube is highlighted. The ENM approach provides the first view of dynamical insights into artificial protein cages, and may open potentially new lines of the further application in nanotechnology.

Symmetries of classical and virtual links

Teruhisa Kadokami

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Abstract: For a classical knot, it is ‘amphicheiral (achiral)’ if it is equivalent to its mirror image, and it is ‘invertible’ if it is oriented and it is equivalent to the orientation-reversed one. These notions can be unified as ‘knot symmetry’, and knot symmetry can be generalized as ‘link symmetry’ for a classical link. The link symmetry has a group structure and it is called the ‘link symmetric group’. For any classical link, to determine the link symmetric group is the fundamental problem. We try to generalize the notion to include ‘periodicity’, and to generalize for the case of virtual links and graphs.

Elements of Khovanov Homology and Khovanov Homotopy

Louis H Kauffman

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Abstract: This is both a research talk and an introductory talk about Khovanov homology. We start with the bracket polynomial model of the Jones polynomial and discuss how Khovanov homology is built from the states of the bracket polynomial by regarding them as

generating a small category. We discuss Bar-Natans tangle-cobordism picture of Khovanov homology, and show how his 4Tube-Relation is obtained naturally in the attempt to make the theory invariant under chain homotopy. We show how the associated Frobenius algebras arise naturally and how all of this fits together. We then examine the question of chain homotopy versus homotopy and show how a judicious use of simplicial theory (the Dold-Kan Theorem) produces natural spaces that behave well stably to give a homotopy theory for Khovanov homology.

Cross-index of a graph

Akio Kawauchi

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Abstract: Given a connected graph G , a topological invariant (called the cross-index) $c_T(G)$ of G taking a value in the set of non-negative integers and ∞ is defined for every tree T . The following properties are given:

- (1) $c_T(G) \neq \infty$ if and only if T is a maximal tree of G , so that $c_T(G) = \infty$ except a finite number of trees T .
- (2) $c_T(G) = 0$ for a maximal tree T of G if and only if $c_T(G) = 0$ for every maximal tree T of G if and only if G is a planar graph.

Two connected graphs G and G' are vertex-congruent if a regular neighborhood of the vertex set $v(G)$ in G is homeomorphic to a regular neighborhood of the vertex set $v(G')$ in G' . Then the same Euler characteristic $\chi(G) = \chi(G')$ is obtained. The following property is also shown:

- (3) For every integer $n > 1$ and some tree T , there are vertex-congruent graphs $G^i (i = 1, 2, \dots, n)$ such that $c_T(G^i) = i$ for every i .

By (2) and (3), it is seen that the cross-index c_T and the Euler characteristic χ are independent topological invariants for connected graphs.

On the Power Domination Number of the Cartesian Product of Graphs

Khee Meng Koh

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Abstract: The power domination problem is to find the minimum number of phase measurement units (PMUs) that will observe the state of the entire electric system. The working principles of these units are based on Ohms law and Kirchoffs current law. Using graph theory terminology, a PMU observes the vertex at which it is placed and its incident edges and their ends. The following observation rules are deduced from the physics laws:

- (a) Any vertex that is incident to an observed edge is observed;
- (b) Any edge joining two observed vertices is observed.

For $k \geq 2$, if a vertex is incident to k edges such that $k - 1$ of these edges are observed, then all k of these edges are observed. Let G be a graph with vertex set $V(G)$. A set $S \subseteq V(G)$ is defined to be a power dominating set (PDS) of G if every vertex and every edge in G are observed by S after applying the above observation rules. The power domination number

$p(G)$ is the minimum cardinality of a PDS of G . In this talk, we give a survey on the existing results about this number for the Cartesian product of graphs.

Star Polynomial and Its Applications to Unimodality and Dominating Uniqueness of Graphs

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Abstract: Let $G = (V(G), E(G))$ be a graph of order n and size m . A subset S of $V(G)$ is a dominating set of $V(G)$ if every vertex in $V(G) \setminus S$ is adjacent to at least one vertex in S . The *dominating number*, denoted by $\gamma(G)$, is the cardinality of a smallest dominating set in G . Let $d(G, i)$ be the number of dominating sets of G of size i . The domination polynomial of G is $D(G, x) = \sum_{i=\gamma(G)}^n d(G, i)x^i$. It was conjectured that all domination polynomials are unimodal but little progress has been made. Two graphs G and H are dominating equivalent if $D(G, x) = D(H, x)$. The set $[G]_{\mathcal{D}} = \{H \mid D(H, x) = D(G, x)\}$ is called the dominating equivalence class determined by G . A graph G is said to be dominating unique if $D(H, x) = D(G, x)$ implies that $H \cong G$. Little is known about dominating uniqueness of graphs too. In this talk, we will look at the role played by \overline{G} in the determination of $D(G, x)$ and its unimodality. In particular, we define a new graph polynomial, called *star polynomial* and use it to obtain infinitely many dominating unique graphs and dominating equivalence classes of graphs. Some conjectures and open problems are also posed.

Ideal coset invariants for surface-links with classical link invariants

Sang Youl Lee

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Abstract: A surface-link is a closed 2-manifold smoothly (or piecewise linearly and locally flatly) embedded in four space. A marked graph diagram is a link diagram possibly with some marked 4-valent vertices. It is known that a surface-link can be described by a marked graph diagram modulo Yoshikawa moves. In this talk, I would like to introduce a method of constructing ideal coset invariants for surface-links by using marked graph diagrams and classical link invariants. This is a joint work with Y. Joung and J. Kim.

The symmetric commutator homology of Link Towers and Homotopy Groups of 3-Manifolds

Fengchun Lei

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Abstract: A link tower is a sequence of links with the structure given by removing the last components. Given a link tower, we prove that there is a chain complex consisting of (non-abelian) groups given by the symmetric commutator subgroup of the normal closures in the link group of the meridians excluding the meridian of the last component with the differential induced by removing the last component. Moreover The homology groups of these naturally constructed chain complexes are isomorphic to the homotopy groups of the manifold M under certain hypothesis. This is joint with Fuquan Fang and Jie Wu.

The Braid Index of Reduced Alternating Braid

Pengyu Liu*

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Abstract: It is well known that any link can be represented by the closure of a braid. The minimum number of strands needed in a braid whose closure represents a given link is called the braid index of the link. Dr. Diao had conjectured that if a link has a reduced alternating diagram which is also the closure of a braid on n strands, then the braid index of the link is exactly n . In this talk we show that this conjecture holds in general indeed. The proof relies on the special formulas of the HOMFLY polynomial for link diagrams in closed braid forms as well as the Morton-Frank-Williams inequality. We show that one of our formulas is equivalent to the expansion derived by F. Jaeger, and our approach provides a combinatorial and shorter proof of his result.

Hyperbolicity of a random link via bridge position

Jiming Ma

Fudan University, China

Abstract: We show that a random link via random bridge position is hyperbolic, this is a joint work with Kazuhiro Ichihara.

Chromatic polynomial, its categorification, and applications to knot theory

Alexander Shumakovitch

Gorge Washington University

Abstract: We start this talk with a brief overview of the construction and main properties of the chromatic graph homology. This homology theory was defined by Helme-Guizon and Rong as a categorification of the chromatic polynomial. We continue by presenting techniques for computing chromatic homology for certain classes of graphs. Finally, we discuss relations between chromatic graph homology and Khovanov homology of knots and show how the former can be used to determine the latter.

Eulerian and Even Face Graph Partial Duals

Metrose Metsidik*

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Abstract: We generalize a well-known theorem, a plane graph is Eulerian if and only if its geometric dual is bipartite, to embedded graphs and partial duals of cellularly embedded graphs, and characterize all Eulerian and all even face graph, a cellularly embedded graph with no odd degree faces, partial duals of a cellularly embedded graph by means of half-edge orientations of its medial graph.

Tunnel number of composite knot

Ruifeng Qiu

East China Normal University, China

Abstract: In this talk, I will talk about some results on the super additivity of tunnel number under connected sum.

The Merino-Welsh Conjecture

Gordon Royle

University of Western Australia, Australia

Abstract: The Merino-Welsh conjecture states that a certain relationship holds between three numerical parameters of a bridgeless, loopless graph G . If we let $a(G)$, $t(G)$ and $c(G)$ denote the number of acyclic orientations, spanning trees and totally cyclic orientations of G (respectively), then the conjecture says that $t(G)$ is less than or equal to the maximum of $a(G)$ and $c(G)$.

In fact, all three of these parameters are evaluations of the Tutte polynomial $T(G; x, y)$ at three points in the xy -plane, namely $a(G) = T(2, 0)$, $t(G) = T(1, 1)$ and $c(G) = T(0, 2)$, and therefore the conjecture can immediately be extended to matroids and viewed as a conjecture about the values of the Tutte polynomial of a matroid on the line segment joining $(0, 2)$ to $(2, 0)$.

In this talk, I will introduce the conjecture including all necessary definitions, state some previous results, outline a proof (joint work with Steve Noble) that the conjecture holds for series-parallel graphs, and describe some extensions of this work due to Seongmin Ok.

Coefficients and non-triviality of Jones polynomials

Alexander Stoimenow

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Abstract: The author plans to discuss the formulas for the edge coefficients of the Jones polynomial in semiadequate diagrams and some applications to non-triviality of the Jones polynomial, odd crossing amphicheiral knots, etc.

On deletion-contraction polynomials for polycyclic chains and spatial graphs

Andrey Vesnin

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Abstract: Many chemically, combinatorially and topologically interesting polynomials could be embedded into a general calculating scheme based on deletion-contraction operations. We discuss two realizations of the scheme. One is done for polycyclic chains of polygons. Explicit expressions are obtained for the Tutte and related polynomials [1]. Polycyclic chains contain some classes of chemically relevant structures, in particular, molecular graphs of unbranched benzenoids hydrocarbons. Another is done for polynomial invariants of spatial graphs [2].

[1] A. Dobrynin, A. Vesnin, On deletion-contraction polynomials for polycyclic chains, MATCH Commun. Math. Comput. Chem., 2014, 72(3), 845-864.

[2] A. Dobrynin, A. Vesnin, On the Yoshinaga polynomial of spatial graphs, Kobe J. Math. 20 (2003) 31-37.

New presentations of a link and virtual link

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Abstract: A new presentation of a link called a simihedron presentation is introduced and an algebraic system on links is constructed. Based on the algebraic system, Reduction Crossing Algorithm is proposed which is used to reduce the number of crossings in a link. For known

unknots, one can transform them into a trivial knot in a polynomial time by applying the algorithm. As a special consequence, Haken's unknot (image courtesy of Cameron Gordon) is given. Similarly, a simihegon presentation of a virtual link is introduced.

On the subgroups of the groups of Brunnian links

Jie Wu

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Abstract: Let $L_n = \{l_1, l_2, \dots, l_n\}$ be a Brunnian link and $G(L_n)$ be its link group. Suppose that R_i is the normal closure of the meridian of l_i in $G(L_n)$, then R_1, R_2, \dots, R_n are normal subgroups of $G(L_n)$. For each $2 \leq m \leq n$, let $X(L_n)_m$ be the homotopy colimit of the classifying spaces $B(G(L_n)/\prod$. Here we studied the geometrical property of $X(L_n)_m$ and issued in an algebraic result, i.e., we proved $\bigcap_{i=1}^m R_i = [R_1, \dots, R_m]_S$, the symmetric commutator subgroup R_1, R_2, \dots, R_n for $1 \leq m \leq n$. This is a joint work with Fengchun Lei and Yu Zhang.

Multi-skein equation Bracket

Zhiqing Yang

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Abstract: The Kauffman Bracket is defined by one skein relation. We modified its definition get a bracket invariant with two skein relations for oriented link diagrams. A simplified version gives a new state mode for the Jones polynomial. As an application, it provides bounds of the number of positive crossings and negative crossings. The general version introduces relation of positive crossing points among the set of all knot diagrams of a fixed knot.

On W_v -Paths in Polyhedral Maps

Dong Ye

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Abstract: The W_v -path conjecture due to Klee and Wolfe states that any two vertices of a simple polytope can be joined by a path that does not revisit any facet. This is equivalent to the well-known Hirsch Conjecture. Klee conjectured even more, namely that the W_v -conjecture is true for all general cell complexes. Klee proved that the W_v -conjecture is true for 3-polytope (3-connected plane graphs). Later, the general W_v -path conjecture was verified for polyhedral maps on the projective plane and torus by Barnette, and on the Klein bottle by Pulapaka and Vince. Recently, however, Santos proved that the Hirsch conjecture is false. In this paper, we show that the W_v -path problem is closely related to both the local connectivity $\kappa_G(x, y)$, and the number of different homotopy classes of (x, y) -paths as well as the number of (x, y) -paths in each homotopy class. For every surface Σ , define a function $f(\Sigma)$ such that if for every graph polyhedrally embedded in Σ and for every pair of vertices x and y in $V(G)$, $\kappa_G(x, y) \geq f(\Sigma)$, then there exists a W_v -path joining x and y . Let $\chi(\Sigma)$ be the Euler characteristic of Σ . In this paper, we show that $f(\Sigma) = 3$ if Σ is the sphere, and for all other surfaces $3 - \tau(\Sigma) \leq f(\Sigma) \leq 9 - 4\chi(\Sigma)$, where $\tau(\Sigma) = \chi(\Sigma)$ if $\chi(\Sigma) < -1$ and 0 otherwise. Further, if x and y are not cofacial, we show that G has at least $\kappa_G(x, y) + 4\chi(\Sigma) - 8$ internally disjoint W_v -paths joining x and y . The bound is sharp for the sphere.

Chromatic polynomials of hypergraphs

Ruixue Zhang*

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Abstract: This report is a survey of results regarding chromatic polynomials of hypergraphs. We shall introduce some recurrence relations for the chromatic polynomials of hypergraphs. Furthermore, some properties with respect to their coefficients and chromatic equivalence also will be presented in this report.

Paths and Cycles in Digraphs and Random Digraphs

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Abstract: Cycle and path properties of graphs have been studied extensively since the early 1950s, and have been a popular field of research within graph theory and computational complexity ever since. Originated from the work and a meta-conjecture of Adrian Bondy, a very important direction in this field is the generalization of various conditions for hamiltonian paths (cycles) to paths (cycles) of many lengths. In this talk, I will report our recent works on some extension properties of paths and cycles in directed graphs (digraphs for short), which imply paths and cycles of many lengths.

We say that a path P in a digraph is extendable, if there exists another path P' with the same starting and ending vertices, such that $V(P) \subseteq V(P')$ and $|P| = |P'| - 1$. A nonempty digraph is called path extendable if every path of it is extendable. Though many extension properties of paths and cycles have been studied, we seem to be the first ones to study path extendability in digraphs. We prove some extremal and degree conditions for path extendable in general digraphs, as well as a result on path extendability in regular tournaments.

We study cycle extendability in bipartite digraphs. In a bipartite digraph, a cycle C is called even extendable if there exists another cycle C' such that $V(C) \subseteq V(C')$ and $|C| = |C'| - 2$. A bipartite digraph is called cycle extendable if it has a cycle and every cycle of it is even extendable. we proved that a bipartite tournament is cycle extendable if and only if it is hamiltonian, unless it belongs to an exceptional class, which can be clearly characterized.

Some existing results related to pancyclicity of bipartite tournaments are then direct consequences of our result.

We also study path extendability and cycle extendability in random tournaments under different models. Some new techniques and concepts are introduced in our work as auxiliary tools. We introduce a contraction operation to transform path non-extendability into cycle non-extendability, which reveals an interesting relation between path extendability and cycle extendability in digraphs. We define the concept of in-out graph of a digraph, which are used to analyse the path-cycle-factors of two intersecting directed cycles.